will involve tensorially contracted products with rotation matrices and with an initial-state density matrix, and the above results are not expected to obtain.

It is simple to see,[4] also, that if two close-lying pairs of trajectories are present, both polarization and $d\sigma/d\Omega$ can exhibit oscillations, but these will be down by a factor s to the power $(\alpha_2-\alpha_1)$ from the dominant terms in the relevant expressions. (Oscillations have been shown to occur, in this situation, in π-N scattering, by Gribov *et al.*[4])

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Unitary Triplets and the Eightfold Way*

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In order to explain the eightfold way, four elementary baryon fields are introduced. Three of them form a unitary triplet and the fourth is a unitary singlet. In this approach, triplets, sextets, etc., are possible multiplets as well as singlets, octets, decuplets, etc. This model has a new quantum number, "hypercharge center." Assuming that the symmetry-breaking interactions transform like components of a triplet, selection rules in the production and decay of the triplets are derived. It is proposed that the isodoublet $\kappa(725)$ along with the isosinglet $z^+(Y=2)$ or $\eta'(Y=0)$ forms a unitary triplet. If the symmetry-breaking interaction transforms like a component of the octet, the following baryon lepton symmetry suggested by Gell-Mann

$$\nu_e \leftrightarrow \text{“}p\text{”}\cos\theta+\text{“}Z\text{”}\sin\theta,$$
$$\nu_\mu \leftrightarrow -\text{“}p\text{”}\sin\theta+\text{“}Z\text{”}\cos\theta$$
$$e^- \leftrightarrow \text{“}n\text{”}\cos\theta'+\text{“}\Lambda'\text{”}\sin\theta',$$
$$\mu^- \leftrightarrow -\text{“}n\text{”}\sin\theta'+\text{“}\Lambda'\text{”}\cos\theta',$$

between four leptons and four elementary baryon fields is shown to be possible.

I. INTRODUCTION

THE success of the broken eightfold way is striking.[1,2] Some have looked for its origin in the bootstrap mechanism.[3,4] However, it is not easy to understand why the bootstrap mechanism prefers the octet scheme of the SU(3) symmetry to other models. The origin of an internal symmetry is most easily understood by introducing elementary fields and a symmetric Lagrangian.

In order to explain the eightfold way, at least four elementary fields are necessary. If we assume the elementary fields are singly charged or neutral, the following two possibilities exist[5]: (a) "p," "n," "Λ," and "Λ'" are elementary where "p" ($B=1$, $Y=1$, $I=\frac{1}{2}$, $Q=1$), "n" ($B=1$, $Y=1$, $I=\frac{1}{2}$, $Q=0$), and "Λ" ($B=1$, $Y=0$, $I=0$, $Q=0$) form a unitary triplet which transforms like 3, and where "Λ'" ($B=1$, $Y=0$, $I=0$, $Q=0$) is a unitary singlet.[6] (b) "n," "p," "Z," and "Λ'" are elementary, where "n," "p," and "Z" ($B=1$, $Y=2$, $I=0$, $Q=1$) form a unitary triplet which transforms like 3*, and where "Λ'" is a unitary singlet.[7]

Here, the fields "p," "n," and "Λ" have no relation to the real p, n, and Λ, which are components of a unitary octet, except that they have the same baryon number, hypercharge, isotopic spin, and charge. At the present time, the particles associated with the fields "p," "n," "Λ," "Z," and "Λ'" have not yet been observed. Their masses must be very large—they may even be infinite.

In this model all stable and unstable particles are considered to be bound states of these elementary particles, and they belong to multiplets corresponding to irreducible representations of SU(3) symmetry. In this article, the mechanism of their dynamical emergence will not be discussed, however.

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[1] M. Gell-Mann, California Institute of Technology Synchrotron Laboratory Report CTSL-20, 1961 (unpublished); Phys. Rev. **125**, 1067 (1962).
[2] Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).
[3] R. E. Cutkosky, Phys. Rev. **131**, 1888 (1963).
[4] R. H. Capps. Phys. Rev. Letters **10**, 312 (1963).
[5] There are two other possibilities: (i) "Ξ^0," "Ξ^-," "Λ," and "Λ'"; and (ii) "Ξ^0," "Ξ^-," "Z^-," and "Λ'." They are equivalent to case (b) and (a) group theoretically.
[6] In the following, the symbols B_1, B_2, B_3, and B are used for "p," "n," "Λ," and "Λ'," respectively, for case (a).
[7] In the following, the symbols B^1, B^2, B^3, and B are used for "n," "p," "Z," and "Λ'," respectively, for case (b).

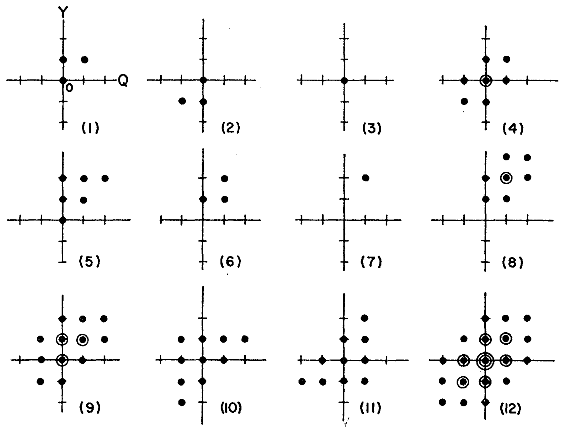

FIG. 1. Some multiplets for case (a): (1) $3(n=1)$, (2) $3^*(n=-1)$, (3) $1(n=0)$, (4) $8(n=0)$, (5) $6(n=2)$, (6) $3^{*\prime}(n=2)$, (7) $1'(n=3)$, (8) $8'(n=3)$, (9) $15(n=1)$, (10) $10(n=0)$, (11) $10^*(n=0)$, (12) $27(n=0)$.

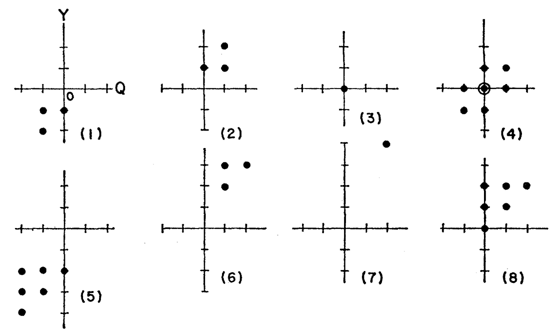

FIG. 2. Some multiplets for case (b): (1) $3(n=1)$, (2) $3^*(n=-1)$, (3) $1(n=0)$, (4) $8(n=0)$, (5) $6'(n=2)$, (6) $3'(n=-2)$, (7) $1'(n=-3)$, (8) $6(n=-1)$.

In the next section, some properties of the multiplets will be discussed and a new quantum number, the "hypercharge-center" will be introduced. Our model can be regarded as a very badly broken SU(4) model.[8] Three additively conserved quantities of the SU(4) group will be the charge, the hypercharge, and the hypercharge-center. In Sec. III, the symmetry-breaking interactions will be introduced, and in Sec. IV a possible baryon-lepton symmetry for case (b) will be discussed. In Sec. V, possible selection rules in the production and decay of unitary triplets will be discussed, introducing unitary triplet mesons and unitary triplet baryon resonances. In Sec. VI, the Gell-Mann–Okubo mass formula[1,9] will be shown to be still valid in our generalized models.

II. POSSIBLE MULTIPLETS AND THE CONSERVATION OF HYPERCHARGE CENTER

Mathematically,[10] the octet mesons Π_j^i can be written as $\bar{B}^iB_j-\frac{1}{3}\delta_j^i\bar{B}^\mu B_\mu$ for case (a), and $\bar{B}_jB^i-\frac{1}{3}\delta_j^i\bar{B}_\mu B^\mu$ for case (b). The octet baryons N_j^i can be written as $B\Pi_j^i$ for both cases. The wave functions of the other multiplets of the eightfold way are constructed from Π_j^i and N_j^i. In the eightfold way, only multiplets constructed from Π_j^i and N_j^i are possible. In our two schemes, other multiplets such as unitary triplets and sextets are possible. Some of these are shown in Figs. 1 and 2. As can be seen from Figs. 1 and 2, $B^i=(B_jB_k-B_kB_j)\bar{B}/\sqrt{2}(i,j,k;$ cyclic 1,2,3). Hence, all representations of case (b) are those of case (a), but some representations of case (a) are not representations of case (b).

From Figs. 1 and 2, we see that the average hypercharge in a multiplet $\langle Y\rangle\equiv(\sum_i Y_i)/$(number of components) is equal to cn (n is an integer) where $c=\frac{2}{3}$ in case (a), $c=-\frac{4}{3}$ in case (b), and $c=0$ in the eightfold way (we shall call $\langle Y\rangle$ the hypercharge center). We also find that all unitary singlets have $Q=\frac{3}{2}cs$ and $Y=3\,cs$, where s is any integer. We can classify irreducible representations by an integer l $(-1\leq l\leq 1)$ defined as $n=3m+l$ (m is any integer).[11] Multiplets that transform like 3, 6*, 15, etc., have $l=1$, while multiplets that transform like 3*, 6, 15*, etc., have $l=-1$. Multiplets that transform like 1, 8, 10, 10*, 27, etc., have $l=0$. Hence, we shall call multiplets with $l=\pm1$ "triplets" and multiplets with $l=0$ "octets." The hypercharge center $\langle Y\rangle$ is additively conserved. For example, $\langle Y\rangle_C=\langle Y\rangle_D=\cdots=\langle Y\rangle_A+\langle Y\rangle_B$ if $A\otimes B=C\oplus D\oplus\cdots$. All particles in a multiplet A have the same hypercharge center $\langle Y\rangle_A$, just as all particles in an isotopic multiplet have the same "charge center," $Y/2$. The hypercharge center is a good quantum number if the hypercharge center of the symmetry-breaking interaction is zero.

III. THE SYMMETRY-BREAKING INTERACTIONS

The symmetry-breaking interactions have to conserve baryon number, isotopic spin, and hypercharge. Candidates can be found from Figs. 1 and 2. For case (a) they are $B_3\bar{B}+B\bar{B}^3$, $\bar{B}^3B_3$, $B_3B_3\bar{B}\bar{B}+BB\bar{B}^3\bar{B}^3$, etc. For case (b) they are $\bar{B}_3B^3$, $[B^1(B^2B^3-B^3B^2)+B^2\times(B^3B^1-B^1B^3)+B^3(B^1B^2-B^2B^1)]\bar{B}_3\bar{B}_3\bar{B}+$h.c., which transforms like $T_{(33)}+T^{(33)}$, etc.[12] In case (b) there are no symmetry-breaking interactions which transform like T^3+T_3.

The success of the first-order broken eightfold way requires both $B_3\bar{B}+B\bar{B}^3$ and $\bar{B}^3B_3$, or $B_3\bar{B}+B\bar{B}^3$, or $\bar{B}^3B_3$ to be elementary for case (a) and $\bar{B}_3B^3$ for case (b). Case (b) also requires $T_{(33)}+T^{(33)}\lesssim T_3^3(\bar{B}_3B^3)$ if $T^{(33)}+T_{(33)}$ is elementary.

In both cases (a) and (b), we can assume that the

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[8] Case (a) of this article has been discussed by P. Tarjanne and V. L. Teplitz, Phys. Rev. Letters **11**, 447 (1963).

[9] S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 947 (1962).

[10] In this section and in the next section, we consider only charge space. Lorentz indices are omitted. The $\bar{B}^i$ ($\bar{B}_i$) are the fields associated with the antiparticles of the particles associated with the fields B_i (B^i).

[11] Multiplets with even m in case (a) are absent in case (b).

[12] The notation $T_{(33)}$ and $T^{(33)}$ was introduced by M. Ikeda, S. Ogawa, and Y. Ohnuki, Progr. Theoret. Phys. (Kyoto) **22**, 715 (1959); **23**, 1073 (1960).

symmetry-breaking interactions transform like $T_3{}^3$, i.e., $\bar{B}^3B_3$ or $\bar{B}_3B^3$. As $T_3{}^3$ is a component of an octet, the hypercharge center of the symmetry-breaking interaction is zero. Thus, the hypercharge center is a good quantum number in this case. In this case, the conservation of the hypercharge center is broken only by weak interactions. Since "octets" and "triplets" have different values of the hypercharge center, a quantity that transforms like a "triplet" has no matrix elements between "octets." Hence, a "triplet" can not be produced in "octet"–"octet" collisions. Of course, two, three, and more "triplets" can be produced associatedly as long as the hypercharge center is conserved.

IV. A POSSIBLE BARYON-LEPTON SYMMETRY

If the masses of "triplets" are very large or infinite, our model with the symmetry-breaking interaction *which transforms like* $T_3{}^3$ agrees with the broken eightfold way except for the fact that we can introduce the following baryon-lepton symmetry between four leptons and four elementary baryon fields for case (b),

$$\nu_e \leftrightarrow \text{“}p\text{”} \cos\theta' + \text{“}Z\text{”} \sin\theta',$$
$$\nu_\mu \leftrightarrow -\text{“}p\text{”} \sin\theta' + \text{“}Z\text{”} \cos\theta',$$
$$e^- \leftrightarrow \text{“}n\text{”} \cos\theta'' + \text{“}\Lambda'\text{”} \sin\theta'',$$
$$\mu^- \leftrightarrow -\text{“}n\text{”} \sin\theta'' + \text{“}\Lambda'\text{”} \cos\theta''.$$

This symmetry was first suggested by Gell-Mann.[13] The weak-interaction current can be written uniquely as

$$J_\alpha = (\bar{\nu}_e O_\alpha e) + (\bar{\nu}_\mu O_\alpha \mu) + [(\text{“}\bar{p}\text{”} \cos\theta + \text{“}\bar{Z}\text{”} \sin\theta) O_\alpha \text{“}n\text{”}] + [(-\text{“}\bar{p}\text{”} \sin\theta + \text{“}\bar{Z}\text{”} \cos\theta) O_\alpha \text{“}\Lambda'\text{”}],$$

where

$$O_\alpha = \gamma_\alpha(1+\gamma_5) \quad \text{and} \quad \theta = \theta' - \theta''.$$

Since $[(-\text{“}\bar{p}\text{”} \sin\theta + \text{“}\bar{Z}\text{”} \cos\theta) O_\alpha \text{“}\Lambda'\text{”}]$ transforms like the components of a unitary triplet (T_2 and T_3), they have no matrix elements between "octets." Thus, the last term of the weak current does not contribute to the leptonic decays of the strongly interacting particles (hadrons). For the same reason, the cross terms

$$[(\text{“}\bar{p}\text{”} \cos\theta + \text{“}\bar{Z}\text{”} \sin\theta) O_\alpha \text{“}n\text{”}]^+ \times [(-\text{“}\bar{p}\text{”} \sin\theta + \text{“}\bar{Z}\text{”} \cos\theta) O_\alpha \text{“}\Lambda'\text{”}] + \text{h.c.}$$

in J^+J+h.c. do not contribute to nonleptonic decays of hadrons.

Only the term $(\text{“}\bar{p}\text{”} O_\alpha \text{“}n\text{”}) \cos\theta$ in J_α contributes to the strangeness-conserving leptonic decays of hadrons. It satisfies $|\Delta I|=1$ and $|\Delta S|=0$. Only the term $(\text{“}\bar{Z}\text{”} O_\alpha \text{“}n\text{”}) \sin\theta$ in J_α contributes to the strangeness-nonconserving leptonic decays of hadrons. It satisfies $|\Delta I|=\frac{1}{2}$ and $|\Delta S|=1$. As is easily seen, the vector parts of these two terms in the weak current are divergenceless in the limit of SU(3) symmetry.

For the strangeness-nonconserving nonleptonic decays of hadrons, the following terms in J^+J contribute:

$$(\text{“}\bar{p}\text{”} O_\alpha \text{“}n\text{”})^+ (\text{“}\bar{Z}\text{”} O_\alpha \text{“}n\text{”}) \sin\theta \cos\theta - (\text{“}\bar{p}\text{”} O_\alpha \text{“}\Lambda'\text{”})^+ (\text{“}\bar{Z}\text{”} O_\alpha \text{“}\Lambda'\text{”}) \sin\theta \cos\theta + \text{h.c.}$$

The first term is a mixture of a $|\Delta I|=\frac{1}{2}$ part and a $|\Delta I|=\frac{3}{2}$ part. The second term is pure $|\Delta I|=\frac{1}{2}$.

The results of this model for leptonic decays of hadrons have been discussed by Cabibbo[14] and by Gell-Mann.[15] A meaningful baryon-lepton symmetry cannot be found for case (a) since there are three neutral baryon fields and one charged field in this case.

V. PRODUCTION AND DECAY OF "TRIPLETS"

In this section let us consider case (a) and let us assume that the symmetry-breaking interaction transforms like the unitary triplet T^3+T_3. Since the hypercharge center of the symmetry-breaking interaction is $\pm\frac{2}{3}$ in this case, the hypercharge center is not a good quantum number. But it is an approximately good quantum number if the symmetry-breaking interaction is small. In this case we have many selection rules in the production and decay processes. Since the hypercharge center is always an integer multiple of $\frac{2}{3}$ in case (a), we have $\sum_{\text{final}} \langle Y \rangle - \sum_{\text{initial}} \langle Y \rangle = \frac{2}{3}n$ (where n is an integer) in any process. This process occurs in the $|n|$th-order violation of SU(3) symmetry if the baryon number, the hypercharge, and the isotopic spin are conserved.

We will show some examples in the following, introducing a hypothetical baryon triplet "N"($Y=1$, $I=\frac{1}{2}$) and "Λ"($Y=0$, $I=0$) and a hypothetical meson triplet κ($Y=1$, $I=\frac{1}{2}$) and z^+($Y=2$, $I=0$) or η'($Y=0$, $I=0$). The hypercharge center of the tripet baryons is $\frac{2}{3}$.

(i) If κ^+, κ°, and z^+ form a unitary triplet [see Fig. 1(6)], their hypercharge center is $\frac{4}{3}$. Since the hypercharge centers of ordinary octets are zero, we obtain the following selection rules:

$K+N \to \kappa+N$	2nd forbidden,
$\to z+Y$	2nd forbidden,
$\to \kappa+\text{“}N\text{”}$	3rd forbidden,
$\to z+\text{“}\Lambda\text{”}$	3rd forbidden,
$\to K+\text{“}N\text{”}$	1st forbidden.
$\pi+N \to \kappa+Y$	2nd forbidden,
$\to z+\Xi$	2nd forbidden,
$\to \kappa+\text{“}\Lambda\text{”}$	3rd forbidden,
$\to K+\text{“}\Lambda\text{”}$	1st forbidden,
$\to \pi+\text{“}N\text{”}$	1st forbidden.
$\bar{K}+N \to \bar{\kappa}+N$	2nd forbidden,
$\to \bar{\kappa}+\text{“}N\text{”}$	1st forbidden,
$\to \bar{K}+\text{“}N\text{”}$	1st forbidden,
$\to \pi+\text{“}\Lambda\text{”}$	1st forbidden.

[13] M. Gell-Mann, Phys. Letters 8, 214 (1964).

[14] N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).

[15] M. Gell-Mann, lecture given at California Institute of Technology, 1963 (unpublished).

The process $\bar{K}+N\to\bar{\kappa}+$"$N$" is third forbidden if we assume a one-octet-meson exchange mechanism for the reaction.

(ii) If κ^+, κ^0, and η' form a unitary triplet [see Fig. 1(1)], their hypercharge center is $\frac{2}{3}$ and we obtain the following selection rules:

$K+N\to\kappa+N$	1st forbidden,
$\to\kappa+$"N"	2nd forbidden.
$\pi+N\to\kappa+Y$	1st forbidden,
$\to\eta'+N$	1st forbidden,
$\to\bar{\eta}'+N$	1st forbidden,
$\to\kappa+$"Λ"	2nd forbidden,
$\to\eta'+$"N"	2nd forbidden,
$\to\bar{\eta}'+$"N"	allowed.
$\bar{K}+N\to\bar{\kappa}+N$	1st forbidden,
$\to\eta'+Y$	1st forbidden,
$\to\bar{\eta}'+Y$	1st forbidden,
$\to\kappa+\Xi$	1st forbidden,
$\to\bar{\kappa}+$"N"	allowed,
$\to\eta'+$"Λ"	2nd forbidden,
$\to\bar{\eta}'+$"Λ"	allowed.

The reactions listed as allowed become 2nd forbidden reactions if we assume a one-octet-meson exchange mechanism for the reaction.

In an interesting letter, Gell-Mann has suggested the possibility that the mesons $\kappa(725)$ are components of an abnormal scalar octet.[16] This may be true. Here, however, we consider another possibility; namely, that the isotopic doublet $\kappa(725)$ forms part of a unitary triplet along with an isotopic singlet neutral meson[17] η' or a single positively charged meson z^+. If the κ are part of a unitary triplet, the small production cross section of κ is easily understood.

The spin and parity of z^+ or η' must be 0^+ or 1^- (if we neglect the possible higher spins) since the κ decay into $K+\pi$. If the mass of z^+ is smaller than $2\mu_K$, z is stable for both strong and electromagnetic interactions and decays only through the weak interactions. If $\mu_z>2\mu_K$, z decays into K^++K^0 are allowed energetically. But it is second forbidden by SU(3) symmetry, and besides, forbidden by charge independence if the spin of z is 0^+. If the spin of z is 0^+, z can decay into $K^++K^0+\gamma$ in order e in the decay amplitude and K^++K^0 in e^2.

Since η' is not equal to its antiparticle $\bar{\eta}'$, $(\eta'+\bar{\eta}')/\sqrt{2}(=\eta_1)$ and $(\eta'-\bar{\eta}')/\sqrt{2}(=\eta_2)$ are the eigenstates of the charge conjugation[18] and the decay occurs from η_1 and η_2. If η_1 is normal scalar (normal vector), η_2 is abnormal scalar (abnormal vector). The decay products of η_1 and η_2 are linear combinations of unitary singlets and the eighth components of unitary octets.[19] The normal scalar η' decays into 2π, but the abnormal scalar η' can decay into $2\pi+\gamma$ (in e) or 4π (in e^2) or 5π if that is possible energetically.[16] If η' is a normal vector particle, it decays into $\pi^++\pi^0+\pi^-$ or into $\pi^++\pi^-$ (in e^2). If η' is an abnormal vector particle, it decays into 4π, $2\pi+\gamma$, and 3π (in e^2). All decays are SU(3) second forbidden. Thus it is possible that isodoublets $\kappa(725)$ and $\bar{\kappa}(725)$ form triplets along with $z^\pm(\mu_z>2\mu_K)$ or with an abnormal vector η' meson and a vector meson ϕ.

At present, $N^{**}(1515)$ and $Y_0^*(1520)$ are considered to form an octet with spin $\frac{3}{2}^-$, along with the yet-to-be-discovered Y_1^* and $\Xi_{1/2}^*$. However, it is possible that N^{**} and Y_0^* form a unitary triplet. Some support for this comes from the relatively narrow reduced widths[20] of N^{**} and Y_0^* compared with the reduced widths of N^*. If we use the expression for the widths,[20] $\Gamma_a=\gamma_a|p^2/(p^2+X^2)|^l p/M_a$ and if we assume[20] $X=350$ MeV/c, the ratio of the reduced widths $\gamma_{N^{**}}/\gamma_{N^*}\approx\frac{1}{3}\sim\frac{1}{6}$. This is a reasonable value since the effect of the broken symmetry on the masses $\Delta m/m$ is about $\frac{1}{5}$.

VI. MASS FORMULAS

If the symmetry breaking interaction transforms like T^3+T_3 in case (a), it does not contribute to mass differences in first order for both "octets" and "triplets" because of the conservation of the hypercharge center. The transformation property of the symmetry-breaking interaction is effectively T_3^3 for mass differences. Therefore, the Gell-Mann–Okubo mass formula,[1,9]

$$m=a+bY+c[Y^2-4I(I+1)],$$

is still valid for both "triplets" and "octets" and for both case (a) and case (b).

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[16] M. Gell-Mann (unpublished).

[17] The possibility that κ^+, κ^0, and η' form a unitary triplet was first suggested by P. Tarjanne and V. L. Teplitz, Phys. Rev. Letters **11**, 447 (1963).

[18] Z. Maki and Y. Ohnuki (unpublished). The author is grateful to them for sending their results before publication.

[19] In the following, η' stands for η_1 or η_2.

[20] S. Glashow and A. H. Rosenfeld, Phys. Rev. Letters **10**, 184 (1963).